

Worksheet 2: matrix algebra

Example 0.12. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Find $\mathbf{A} + \mathbf{B}$, $\mathbf{A} - \mathbf{B}$, $3\mathbf{B}$ and $\mathbf{A} + 3\mathbf{B}$.

Example 0.13. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 0 \end{bmatrix}.$$

Find \mathbf{AB} and \mathbf{BA} . Are they the same?

Example 0.14. Let

$$\mathbf{A} = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \end{bmatrix}.$$

Find \mathbf{AB} . Is \mathbf{BA} defined?

Example 0.15. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Find \mathbf{A}^3 and \mathbf{B}^3 . What are \mathbf{A}^k and \mathbf{B}^k for $k > 3$?

Example 0.16. Find the transpose of the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 4 \\ 4 & 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

Example 0.17. Verify that $\mathbf{A} = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix}$ are inverses of each other and then use this fact to solve the matrix equation $\mathbf{Ax} = \mathbf{b}$ for $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

Example 0.18. Use the empirical rule to find the inverse of

$$\mathbf{A} = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix}$$

Example 0.19. Find the inverse of the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix},$$

if its exists.